International Journal of Theoretical Physics, Vol. 27, No. 3, 1988

Satellite Test for Dragging of Inertial Frames

Arnold Rosenblum[1] and Manfred Treber[1,2]

Received August 3, 1987

It is shown that it is possible by using the lack of synchronization of clocks by light signal synchronization in elliptical orbits to test for the dragging of inertial frames in Einstein's theory of general relativity. Possible experiments are discussed.

The vast improvement in clock accuracy, the recent launching of satellites like the Canadian Anik I, and the planned launching of American satellites in the immediate future containing clocks and/or transponders makes possible highly accurate clock synchronization on a world-wide basis.

In the special theory of relativity, a fundamental procedure is the synchronization of standard clocks at rest in any inertial system using light signals (Einstein, 1905; Stachel, 1980). Then, in general, the synchronization between the clocks at the end points of a curve depends on the particular curve chosen, i.e., the synchronization is path-dependent (Landau and Lifshitz, 1951; Rindler, 1969; Cohen and Moses, 1977; Cohen *et al.*, 1983).

We will show that for clocks lying on a circle, the lack of synchronization due to the dragging of inertial frames is within the realm of current experimental physics.

We first review the method of synchronization using closely spaced clocks (Landau and Lifshitz, 1951; Cohen *et al.*, 1984). For a general stationary metric the "time differential" in the noninertial reference frame is not exact. This is most easily seen by changing the form of the metric line element

$$\begin{aligned} ds^2 &= g_{\alpha\beta}\, dx^\alpha\, dx^\beta \\ &= g_{00}(dx^0)^2 + 2g_{0i}\, dx^0\, dx^i + g_{ij}\, dx^i\, dx^j \end{aligned} \tag{1}$$

[1]International Institute of Theoretical Physics, Utah State University, Logan, Utah 84322.
[2]Institute for Theoretical Physics, University of Karlsrhue, Karlsruhe, West Germany.

to

$$ds^2 = g_{00}(dt + g_{00}^{-1} g_{0i}\, dx^i)^2 + (g_{ij} - g_{00}^{-1} g_{0i} g_{0j})\, dx^i\, dx^0 \tag{2}$$

The last term in equation (2) will be recognized as the spatial metric, which is, of course, a local concept (Landau and Lifshitz, 1951; Cohen *et al.*, 1984). Hence, everywhere on the spatial surface defined by the spatial metric, the "time differential"

$$\text{“}dt\text{”} = dt + g_{00}^{-1} g_{0i}\, dx^i \tag{3}$$

vanishes. Despite the vanishing of "dt" everywhere on the spatial surface, the integral of "dt" does not vanish (except in the special case where the exterior derivative of "dt" vanishes). Thus, integration around a closed contour ∂C bounding a region C gives the global result

$$\Delta t = \int_{\partial C} g_{00}^{-1} g_{0i}\, dx^i \tag{4}$$

The use of the generalized Stokes theorem transforms this into a surface integral

$$\Delta t = \int_c d(g_{00}^{-1} g_{0i}\, dx^i) \tag{5}$$

where d denotes the exterior derivative.

For a uniformly rotating frame in Minkowski space, the line element of equation (2) becomes

$$\begin{aligned} ds^2 = {} & \gamma^{-2} c^2 (dt - \gamma^2 c^{-2} \omega r - \gamma^2 c^{-2} \omega r^2 \sin^2\theta\, d\phi)^2 \\ & + dr^2 + r^2\, d\theta^2 + r^2 \gamma^2 \sin^2\theta\, d\phi^2 \end{aligned} \tag{6}$$

where $\gamma^{-2} = 1 - r^2 \omega^2 c^{-2}$, since, using the line element (6), we obtain (assuming $\omega r c^{-1} \ll 1$)

$$\Delta t = \pm(2\omega)/c^2 A \tag{7}$$

where A is the projected area of the contour on a plane perpendicular to the axis of rotation, with the plus or minus sign holding according to whether we go around the contour in the direction of or opposite to the direction of rotation.

To discuss the effect of dragging of inertial frames, we make use of the Brill and Cohen solution (Brill and Cohen, 1966). They have shown that in the equatorial plane of a slowly rotating massive object the metric can be written

$$-d\tau^2 = -A^2\, dt^2 + B^2\, dr^2 + r^2(d\phi - \Omega\, dt)^2$$

where

$$A^2 = B^{-2} = 1 - 2m/r, \qquad \Omega = 2J/r^3 - \omega_0 \tag{8}$$

with ω_0 the rotation rate of an inertial frame as measured by an observer at infinity, and with J the body's angular momentum, $J = KM_+R_+^2\omega$ ($K \simeq 1$). The synchronization gap expressed in terms of the proper time for orbiting satellites in elliptical equatorial orbit is

$$\Delta S = 2\pi r^2 \omega_0[1 + \tfrac{3}{2}M/r - (2KM/r)(R_+/r)^2\omega/\omega_0] \tag{9}$$

where the first term in the brackets is the special relativistic one, the second term is similar to the gravitational redshift, and the third term arises from the dragging of inertial frames.

For experimental purposes, it is clearly better to eliminate the lower order terms in expression (9). To do this, we send up clocks in satellites in both elliptical equivalent geosynchronous and antigeosynchronous orbits. We synchronize using light signals, first the clocks in the geosynchronous orbits and then by using light signals going the other way for the clocks in antigeosynchronous orbits. The simple addition of the synchronization gap of both the geosynchronous and antigeosynchronous orbits will isolate the effects of dragging of inertial frames, and with possible clock accuracies of one part in 10^{18} and stability over a 2-year period in the next 5 years, this leads to the possibility of experimental determination of the dragging of inertial frames (Dehmelt, 1985). Another possibility is the use of the millisecond pulsar as a clock. One revolution in geosynchronous orbit leads to a synchronization gap of approximately 5.76×10^{-16} sec, which is an order of magnitude better than the use of circular orbits. Research is now under way to tackle detailed experimental problems.

To summarize, advances in clock technology lead to the possibility of new tests of general relativity.

ACKNOWLEDGMENTS

We would like to thank Profs. Jurgen Ehlers, Wolfgang Rindler, and Kip Thorne for helpful critical remarks.

REFERENCES

Brill, D., and Cohen, J. M. (1966). *Physical Review* **143**, 1011.
Cohen, J. M., and Moses, H. E. (1977). *Physical Review Letters*, **39**, 1641.

Cohen, J. M., Moses, H. E., and Rosenblum, A. (1983). *Physical Review Letters*, **J1**, 1501.

Cohen, J. M., Moses, H. E., and Rosenblum, A. (1984). *Classical and Quantum Gravity*, L57–L59.

Dehmelt, H. (1985). *Annales de Physique Française* **10**, 777.

Einstein, A. (1905). *Annalen der Physik*, **17**, 871.

Landau, L. D., and Lifshitz, E. M. (1951). *The Classical Theory of Fields*, Addison-Wesley, Cambridge.

Rindler, W. (1969). *Essential Relativity*, Van Nostrand, New York.

Stachel, J. (1980). In *General Relativity and Gravitation*, A. Held, ed., Vol. 1, p. 1, Plenum Press, New York.